R0061

Sub. Code
511101

M.Sc. DEGREE EXAMINATION, NOVEMBER – 2023

First Semester

Mathematics

GROUPS AND RINGS

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 1 = 10)$

Answer **all** the following objective questions by choosing the correct option.

1. Let G be the group of all 2×2 matrices $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ where $ad - bc \neq 0$ and a, b, c, d are integers modulo 3, relative to matrix multiplication. Then which of the following is $O(G)$? (CO1, K1)

   (a) 10 (b) 48

   (c) 28 (d) 30

2. Which of the following group is cyclic? (CO1, K1)

   (a) D_n -Dihedral group

   (b) U_9 -The integers relatively prime to n under multiplication mod a

   (c) S_3 -Symmetric group of degree 3

   (d) $GL_2(\mathbb{R})$-The set of all 2×2 invertible matrices over real numbers

3. Which of the following Statement is correct? (CO2, K2)

(a) Every subgroup of an non-abelian group is normal

(b) If N and M are normal subgroups of G, then NM is need not be a normal subgroup of G.

(c) Commutator subgroup of G is normal in G

(d) If H is a subgroup of G and N is a normal subgroups of G, then $H \cup N$ is a normal subgroup of H.

4. The number of group homomorphisms from S_3 to $Z/6Z$? (CO3, K3)

(a) 5 (b) 6

(c) 3 (d) 2

5. Let $\alpha = (1,3,5,7,9,11)$ and $\beta = (2, 4, 6, 8)$ be two permutations in S_{100}. What is the order of $\alpha\beta$? (CO3, K3)

(a) 4 (b) 6

(c) 12 (d) 100

6. Let G be a simple group of order 168. Which of the following number of subgroups of G of order 7? (CO3, K4)

(a) 1 (b) 8

(c) 7 (d) 28

7. Which of the following commutative ring is integral domain? (CO3, K3)

(a) The ring of integers mod 6

(b) A product of two non-zero commutative ring

(c) The quotient ring $\mathbb{Z}[x]/x^2 - n^2$ for any integer n

(d) The ring of integers mod p, where p is a prime number

8. Let $\mathbb{R}$ be the ring of all real-valued continuous functions on the closed unit interval. Then pick out the maximal ideal of $\mathbb{R}$. (CO3, K4)

(a) $M = \{f(x) \in \mathbb{R} / f(1) = 1\}$

(b) $M = \{f(x) \in \mathbb{R} / f\left(\frac{1}{2}\right) = 1\}$

(c) $M = \{f(x) \in \mathbb{R} / f\left(\frac{1}{2}\right) = 0\}$

(d) $M = \{f(x) \in \mathbb{R} / f\left(\frac{1}{3}\right) = \frac{1}{3}\}$

9. Which of the following is Euclidean ring? (CO5, K5)

(a) $Z[\sqrt{-5}] = \{a + b\sqrt{-5} / a$ and b are integers$\}$

(b) The ring of Gaussian integers

(c) The ring $A = \mathbb{R}[X, Y] / X^2 + Y^2 = 1$

(d) $Z[\sqrt{-19}] = \{a + b\sqrt{-19} / a$ and b are integers$\}$

10. If $f(x)$ is in $F(x)$, where F is the field of integers mod P, P a prime number, and $f(x)$ is irreducible over F of degree n. Then $F[x]/(f(x))$ is a field with ———— elements.

(CO5, K6)

(a) n (b) P

(c) n^P (d) P^n

Part B $(5 \times 5 = 25)$

Answer **all** the questions not more than 500 words each.

11. (a) If G is a finite group and H and K are finite subgroups of G of order $O(H)$ and $O(K)$ respectively, then prove that $O(HK) = \frac{O(H)\,O(K)}{O(H \cap K)}$ (CO1, K2)

Or

(b) Let U_n denote the integers relatively prime to n under multiplication mod n. Show that U_{17} is a cyclic group. What are all its generators? (CO2, K3)

12. (a) If ϕ is a homomorphism of G into $\overline{G}$ with Kernel K, then prove that K is a normal subgroup of G. (CO3, K4)

Or

(b) For any group G, prove that the commutator subgroup G^1 is a characteristic subgroup of G. (CO4, K4)

13. (a) State and prove third part of Sylow's theorem. (CO5, K5)

Or

(b) Let A, B be cyclic groups of order m and n respectively. Prove that $A \times B$ is cyclic if and only if m and n are relatively prime. (CO5, K5)

14. (a) Prove a finite integral domain is a field. (CO5, K5)

Or

(b) If F is a field, prove its only ideal are (0) and F itself. (CO5, K6)

15. (a) Let $\mathbb{R}$ be a Euclidean ring and $a, b \in \mathbb{R}$. If $b \neq 0$ is not a unit in $\mathbb{R}$, then prove that $d(a) < d(ab)$. (CO5, K6)

Or

(b) Prove that if an ideal U of a ring $\mathbb{R}$ contains a unit of $\mathbb{R}$, then prove that $U = \mathbb{R}$. (CO5, K6)

Part C $(5 \times 8 = 40)$

Answer **all** the questions not more than 1000 words each.

16. (a) (i) Show that if every element of the group G is its own inverse, then G is abelian. (CO5, K6)

(ii) If G is a group of even order, prove it has an element $a \neq e$ satisfying $a^2 = e$.

(iii) Let G be the group of all non-zero complex number $a + bi$; a, b are real and not both zero, and let $H = \{a + ib \in G / a^2 + b^2 = 1\}$. Verify that H is a subgroup of G

Or

(b) (i) Prove that HK is a subgroup of G if and only if $HK = KH$. (CO5, K4)

(ii) Prove that any subgroup of a cyclic group is itself a cyclic group.

(iii) If G is a finite group and $a \in G$, then prove that $O(a)/O(G)$.

17. (a) Let ϕ be a homomorphism of G onto $\overline{G}$ with kernal K. Then prove that $G/K \approx \overline{G}$. (CO5, K6)

Or

(b) State and prove Cayley theorem. (CO5, K6)

18. (a) (i) State and prove Cauchy theorem. (CO5, K6)

(ii) State and prove Second part of Sylow's theorem.

Or

(b) Let G be a group and suppose that G is the internal direct product of $N_1, \ldots\ldots N_n$. Let $T = N_1 \times N_2 \times \ldots\ldots \times N_n$. Then prove that G and T are isomorphic. (CO5, K5)

19. (a) (i) Let R be a commutative ring with unit element whose only ideals are (o) and R itself. Then prove that R is a field. (CO5, K6)

(ii) If R is a commutative ring with unit element and M is an ideal of R, then prove that M is maximal ideal of R if and only if R/M is field.

Or

(b) (i) If D is an integral domain and D is of finite characteristic, then prove that the characteristic of D is a prime number. (CO5, K5)

(ii) If U, V are ideals of R, let $U + V = \{u + v / u \in U, v \in V\}$. Prove that $U + V$ is also an ideal.

(iii) Write the statement of the Pigeonhole principle.

20. (a) (i) Show that the ideal $A = (a_o)$ is a maximal ideal of the Euclidean ring R if and only if a_o is a prime element of R. (CO5, K6)

(ii) Prove that the domain of Gaussian integers $J[i]$ is a Euclidean ring.

Or

(b) (i) Prove that $x^2 + x + 4$ is irreducible over F, the field of integers mod 11 and prove directly that $F[x]/x^2 + x + 4$ is a field having 121 elements. (CO5, K6)

(ii) If P is a prime number, prove that the polynomial $x^n - P$ is irreducible over the rationals.

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R0062

Sub. Code
511102

M.Sc. DEGREE EXAMINATION, NOVEMBER – 2023

First Semester

Mathematics

REAL ANALYSIS – I

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 1 = 10)$

Answer the following objective questions by choosing the correct option.

1. Every infinite subset of a countable set A is ______________. (CO1, K2)

   (a) Countable (b) Uncountable

   (c) Compact (d) Complete

2. Let A be the set of real numbers x such that $0 < x \leq 1$. For every $x \in A$, let E_x be the set of real numbers y such that $0 < y < x$. Then ______________. (CO1, K2)

   (a) $\bigcap_{x \in A} E_x$ is non empty

   (b) $\bigcup_{x \in A} E_x = E_1$

   (c) $E_x \supset E_z$

   (d) $\bigcap_{x \in A} E_x = E_1$

3. If $s_n = i^n$, the sequence $\{s_n\}$ is ______________. (CO2, K3)

(a) Divergent, unbounded and infinite range

(b) Converges, bounded and infinite range

(c) Converges, bounded and finite range

(d) Divergent, bounded and finite range

4. A metric space in which every Cauchy sequence converges is said to be ______________. (CO2, K3)

(a) Compact (b) Continuous

(c) Complete (d) None of these

5. If the sequence is convergent then ______________.

(CO3, K4)

(a) It has two limits

(b) It is bounded

(c) It is bounded above but may not be bounded below

(d) It is bounded below but may not be bounded above

6. Every Cauchy sequence has a ______________. (CO3, K4)

(a) Convergent subsequence

(b) Increasing subsequence

(c) Decreasing subsequence

(d) Positive subsequence

7. A number L is called limit of the function f when x approaches to c if for all $\varepsilon > 0$, there exists $\delta > 0$ such that ______________ $0 < |x - c| < \delta$. (CO4, K4)

(a) $|f(x) - L| > \varepsilon$ (b) $|f(x) - L| < \varepsilon$

(c) $|f(x) - L| \leq \varepsilon$ (d) $|f(x) - L| \geq \varepsilon$

8. If $\lim_{x \to c} f(x) = L$, then ______________ sequence $\{x_n\}$ such that $x_n \to c$ when $n \to \infty$, one has $\lim_{n \to \infty} f(x) = L$. (CO4, K5)

(a) For some

(b) For every

(c) For every subsequence

(d) For some subsequence

9. Suppose f and g are defined on [a, b] and are differentiable at a point $x \in$ [a, b]. Then f + g is differentiable and (CO5, K6)

(a) $(f+g)'(x) = f'(x) + g'(x)$

(b) $(f+g)'(x) = f(x) + g(x)$

(c) $(f+g)'(x) = f'(x) + g'(x)$

(d) $(f+g)'(x) = f(x) + g(x)$

10. Let f be defined on $[a,b]$. If f is differentiable at a point $x \in [a,b]$. then ______________. (CO5, K6)

(a) f is continuous at x

(b) f is discontinuous at x

(c) bounded

(d) unbounded

Part B $(5 \times 5 = 25)$

Answer **all** the questions not more than 500 words each.

11. (a) Show that compact subsets of metric spaces are closed. (CO1, K2)

Or

(b) (i) Prove that every neighborhood is an open set.

(ii) Prove that a set E is open if and only if its complement is closed. (CO1, K2)

12. (a) Prove that the following (CO2, K3)

(i) If $\{p_n\}$is a sequence in a compact metric space X, then some subsequence of $\{p_n\}$ converges to a point of X.

(ii) Even bounded sequence in R^k contains a convergent subsequence.

Or

(b) Let $\{s_n\}$ be a sequence of real numbers. Let E and $S*$ be the lower limit of$\{s_n\}$. Then prose S* has the following properties. (CO2, K3)

(i) $s* \in E$

(ii) If $x > s*$, there is an integer N such that $n \geq N$ implies $s_n < x$.

13. (a) For any sequence $\{c_n\}$ of positive numbers, prove that $\lim_{n\to\infty} \inf f \frac{c_{n+1}}{c_n} \leq \lim_{n\to\infty} inf\, f^n \sqrt{c_n}$ and $\lim_{n\to\infty} \sup^n \sqrt{c_n} \leq \lim_{n\to\infty} \sup \frac{c_{n+1}}{c_n}$. (CO3, K4)

Or

(b) Suppose (CO3, K4)

(i) The partial sums A_n of $\sum a_n$ form a bounded sequences;

(ii) $b_0 \geq b_1 \geq b_2 \geq \ldots,$

(iii) $\lim_{n\to\infty} b_n = 0$

Then prove that $\sum a_n b_n$ converges.

14. (a) Prove that a mapping f of a metric space X into a metric space Y is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set in V in Y.

(CO4, K5)

Or

(b) Suppose f is a continuous mapping of a compact metric space into a metric space Y. Then prove that $f(X)$ is compact. (CO4, K5)

15. (a) Suppose f is continuous on $[a,b], f'(x)$ exists at some point $x \in [a,b], g$ is defined on an interval I which contains the rangeof f, and y is differentiable at the point $f(x)$. If $h(t) = g(f(t)), (a \le t \le b)$, then prove that h is differentiable at x and $h'(x) = g'(f(x))f'(x)$.

(CO5, K6)

Or

(b) Suppose f is a continuous mapping of $[a,b]$ into R^k and f is differentiable in (a,b). Prove that there exists $x \in (a,b)$ such that $|f(b) - f(a)| \le (b-a)|f'(x)|$.

(CO5, K6)

Part C (5 × 8 = 40)

Answer **all** the questions not more than 1000 words each.

16. (a) Suppose $K \subset Y \subset X$. Prove that K is compact relative to X if and only if K is compact relative to Y.

(CO1, K2)

Or

(b) Show that even k-cell is compact. (CO1, K2)

17. (a) (i) If $p > 0$,then prove that $\lim_{n\to\infty} \frac{1}{n^p} = 0$ (CO2, K3)

(ii) If $p > 0$, then prove that $\lim_{n\to\infty} \sqrt[n]{n} = 1$

(iii) Prove $\lim_{n\to\infty} \sqrt[n]{n} = 1$

(iv) If $p > 0$ and α is real, then prove that $\lim_{n\to\infty} \frac{n^\alpha}{(1+p)^n} = 0$

(v) If $|x| < 1$, then prove that $\lim_{n\to\infty} x^n = 0$.

Or

(b) Show that if $p > 1, \sum_{n=2}^{\infty} \frac{1}{n(\log n)^p}$ converges: If $p \leq 1$, the series diverges. (CO2, K3)

18. (a) Let $\sum a_n$ be a series of real numbers which converges, but not absolutely. Suppose $-\infty \leq \alpha \leq \beta \leq \infty$. Prove that there exists a rearrangement $\sum a_n'$ with partial sums s_n' such that $\lim_{n\to\infty} in\, fs_n' = \alpha, \lim_{n\to\infty} \inf s'_n = \beta$. (CO3, K3)

Or

(b) Suppose (CO3, K4)

(i) $\sum_{n=0}^{\infty} a_n$ converges absolutely,

(ii) $\sum_{n=0}^{\infty} a_n = A$

(iii) $\sum_{n=0}^{\infty} b_n = B$

(iv) $c_n = \sum_{k=0}^{n} a_k b_{n-k} \quad (n = 0,1,2,\ldots)$

Then prove that $\sum_{n=0}^{\infty} c_n = AB$

19. (a) Let f be a continuous mapping of a compact metric space X into a metric space Y. Prove that f is uniformly continuous on X. (CO4, K4)

Or

(b) Let E be a noncompact set in R^1. Then prove the following. (CO4, K5)

(i) there exists a continuous function on E which is not bounded:

(ii) there exists a continuous and bounded function on E which has no maximum.

If, in addition, E is bounded, then

(iii) there exists a continuous function on E which is not uniformly continuous.

20. (a) Suppose f and g are real and differentiable in (a,b) and $g'(x) \neq 0$ for all $x \in (a,b)$, where $-\infty \leq a < b \leq +\infty$. Suppose $\frac{f'(x)}{g'(x)} \to A$ as $x \to a$. If $f(x) \to 0$ and $g(x) \to 0$ as $x \to a$, or if $g(x) \to +\infty$ as $x \to a$, prove that $\frac{f(x)}{g(x)} \to A$ as $x \to a$. (CO5, K6)

Or

(b) If f and g are continuous real functions on $[a,b]$ which are differentiable in (a,b), prove that there is a point $x \in (a,b)$, at which $[f(b) - f(a)]g'(x) = [g(b) - g(a)]f'(x)$. (CO5, K6)

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R0063

Sub. Code
511103

M.Sc. DEGREE EXAMINATION, NOVEMBER – 2023

First Semester

Mathematics

ORDINARY DIFFERENTIAL EQUATIONS

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A (10 × 1 = 10)

Answer **all** the following objective questions by choosing the correct option.

1. Find all solutions of the differential equation $y'' - 4y = 0$ (CO1, K1)

   (a) $\phi(x) = c_1 e^{2x} + c_2 e^{-2x}$ where c_1, c_2 are any constants

   (b) $\phi(x) = c_1 e^{x} + c_2 e^{-x}$ where c_1, c_2 are any constants

   (c) $\phi(x) = c_1 e^{2x} + c_2 e^{-x}$ where c_1, c_2 are any constants

   (d) $\phi(x) = c_1 e^{x} + c_2 e^{-2x}$ where c_1, c_2 are any constants

2. Let W be the Wronskian of two linearly independent solutions of ODE $2y'' + y' + t^2 y = 0$; $t \in R$. Then, for all t, there exists a constant $C \in R$ such that $W(t)$ is (CO1, K2)

   (a) Ce^{-t} (b) $Ce^{\frac{t}{2}}$

   (c) Ce^{2t} (d) Ce^{-2t}

3. The functions ϕ_1, ϕ_2 defined below exist for $-\infty < x < \infty$. Determine which functions are linearly dependent here (CO2, K4)

(i) $\phi_1(x) = x, \phi_2(x) = e^{rx}$ where r is a complex constant

(ii) $\phi_1(x) = x^2, \phi_2(x) = 5x^2$

(iii) $\phi_1(x) = x, \phi_2(x) = |x|$

(a) Only (i) is linearly dependent

(b) Only (ii) is linearly dependent

(c) All the above are linearly dependent

(d) None of the above are linearly dependent

4. If $J[y] = \int_1^2 \left(y'^2 + 2yy' + y^2\right) dx$, $y(1) = 1$ and $y(2)$ is arbitrary then the external is (CO2, K5)

(a) e^{x-1} (b) e^{x+1}

(c) e^{1-x} (d) e^{-x-1}

5. The differential equation $\frac{dy}{dx} = 60(y^2)^{\frac{1}{5}}; x > 0,\ y(0) = 0$ has (CO3, K6)

(a) A unique solution

(b) Two solution

(c) No solution

(d) Infinite number os solutions

6. Consider the second order differential equation $y'' + y' - 2y = \sin x$. Find the roots of the auxiliary equation of the given ODE? (CO3, K1)

(a) –2 and 1 (b) –2 and –1

(c) 2 and 1 (d) 2 and – 1

7. Consider the ODE $y''+P(x)y'+Q(x)y=0$ where P and Q are smooth functions. Let y_1 and y_2 be any two solutions of the ODE. Let $W(x)$ be the corresponding Wronskian. Then which of the following is always true? (CO4, K2)

(a) If y_1 and y_2 are linearly independent then $\exists x_1, x_2$ such that $W(x_1)=0$ and $W(X_2)\neq 0$

(b) If y_1 and y_2 are linearly independent $W(x)=0\,\forall x$

(c) If y_1 and y_2 are linearly dependent then $W(x)\neq 0\,\forall x$

(d) If y_1 and y_2 are linearly independent then $W(x)\neq 0\,\forall x$

8. Find the basis for the solutions of the second order differential equation $y''-\frac{2}{x^2}y=x,(0<x<\infty)$? (CO4, K3)

(a) x^2 and x^{-1} (b) x^{-2} and x^{-1}

(c) x^{-2} and x^1 (d) x^2 and x^1

9. Find the solution ϕ of $y''=1+(y')^2$ which satisfies $\phi(0)=0, \phi'(0)=0$? (CO5, K5)

(a) $\phi(x)=-\log(\cos x),\ (-\frac{\pi}{2}<x<\frac{\pi}{2})$

(b) $\phi(x)=\log(\cos x),\ (-\frac{\pi}{2}<x<\frac{\pi}{2})$

(c) $\phi(x)=-\log(\sin x),\ (-\frac{\pi}{2}<x<\frac{\pi}{2})$

(d) $\phi(x)=\log(\sin x),\ (-\frac{\pi}{2}<x<\frac{\pi}{2})$

10. Consider the differential equation $(x-1)y''+xy'+\frac{1}{x}y=0$ then (CO5, K6)

(a) $x=1$ is the only singular point

(b) $x=0$ is the only singular point

(c) Both $x=0$ and $x=1$ are singular points

(d) Neither $x=0$ nor $x=1$ are singular points

Part B $(5 \times 5 = 25)$

Answer **all** questions, not more than 500 words each.

11. (a) If ϕ_1, ϕ_2 are two solutions of $L(y)=0$ on an interval I containing a point x_0, then prove that $W(\phi_1,\phi_2)(x)=e^{-a_1(x-x_0)}W(\phi_1,\phi_2)(x_0)$. (CO1, K2)

Or

(b) Let $\phi_1,...,\phi_n$ be n linearly independent solutions of $L(y)=0$ on an interval *I*. If $c_1,...,c_n$ are any constants, then show that $\phi=c_1\phi_1+....+c_n\phi_n$ is a solution, and every solution can be represented in that form. (CO1, K2)

12. (a) Find all the solutions of the second order differential equation $y''-\frac{2}{x^2}y=x, (0<x<\infty)$. (CO2, K3)

Or

(b) Discuss about the reduction of the order of a homogeneous equation. (CO2, K4)

13. (a) Write a short note on the derivation of second order equations with regular and singular points. (CO3, K6)

Or

(b) Formulate the solution of the Bessel's function of order α of the first kind J_α. (CO3, K5)

14. (a) Suppose S is either a rectangle $|x-x_0|\le a, |y-y_0|\le b, (a, b>0)$, or a strip $|x-x_0|\le a, |y|<\infty, (a>0)$, and that f is a real-valued function defined on S such that $\frac{\partial f}{\partial y}$ exists, is continuous on S, and $\left|\frac{\partial f}{\partial y}(x, y)\right|\le K, ((x, y) \text{ in } S)$, for some $K>0$. Show that f satisfies a Lipschitz condition on S with Lipschitz constant K. (CO4, K3)

Or

(b) Prove that a function ϕ is a solution of the initial value problem $y'=f(x,y),\ y(x_0)=y_0$, on an interval I if and only if it is a solution of the integral equation $y=y_0+\int_{x_0}^{x} f(t,y)\,dt$ on I. (CO4, K2)

15. (a) Suppose f is a real-valued continuous function on the plane $|x|<\infty, |y|<\infty$, which satisfies a Lipschitz condition on each strip $S_a : |x|\le a, |y|<\infty$, where a is any positive number. The lipschitz constant K_a for f in S_a may depend on a. Prove that every initial value problem $y'=f(x,y),\ y(x_0)=y_0$, has a solution which exists for all real x. (CO5, K5)

Or

(b) Let f be continuous and satisfy a Lipschitz condition on R. Let the $g_k(k=1,2,..)$ be continuous on R and satisfy $|f(x, y)-g_k(x,y)|\leq\in_k$, (all (x,y) in R), for some constant $\in_k\to 0(k\to\infty)$, and let $y_k\to y_0(k\to\infty)$. If ψ_k is a solution of $y'=g_k(x,y)$, $y(x_0)=y_k$ on an interval I containing x_0 and ϕ is the solution of $y'=f(x,y), y(x_0)=y_0$ on I, then show that $\psi_k(x)\to\phi(x)$ on *I*. (CO5, K6)

Part C (5 × 8 = 40)

Answer **all** the questions, not more than 1000 words each.

16. (a) (i) Show that the two solutions ϕ_1,ϕ_2 of $L(y)=0$ are linearly independent on an interval I if, and only if, $W(\phi_1,\phi_2)(x)\neq 0$ for all x in I.

(ii) Let ϕ_1,ϕ_2 be two solutions of $L(y)=0$ on an interval I, and let x_0 be any point in I. Verify that ϕ_1, ϕ_2 are linearly independent on I if and only if $W(\phi_1,\phi_2)(x_0)\neq 0$. (CO1, K2)

Or

(b) State and prove the existence theorem for the linear equations with constant coefficients. (CO1, K4)

17. (a) Let ϕ_1 be a solution of $L(y)=0$ on an interval I, and suppose $\phi_1(x)\neq 0$ on I. If $v_2,..,v_n$ is any basis on I for the solutions of the linear equation $\phi_1 v^{(n-1)}+...+[n\phi_1^{(n-1)}+(n-1)\phi_1^{(n-2)}+...+a_{n-1}\phi_1]v=0$ of order $n-1$ and if $v_k=u_k'$, $(k=2,...,n)$, then prove that $\phi_1, u_2\phi_1,...,u_n\phi_1$ is a basis for the solutions of $L(y)=0$ on *I*. (CO2, K3)

Or

(b) Generate a detailed derivation for the solutions of the second order linear equations with analytic coefficients (Legendre equation). (CO2, K6)

18. (a) Derive the Bessel's function of zero order of the second kind K_0. (CO3, K5)

Or

(b) Consider the second order differential equation $x^2 y''+xy'+y=0$ for $x \neq 0$. Apply Euler's method to find the solution of $L(y)=0$. (CO3, K3)

19. (a) Let M, N be two real-valued functions which have continuous first partial derivatives on some rectangle $R:|x-x_0|\leq a, |y-y_0|\leq b$. Prove that the equation $M(x, y)+N(x,y)y'=0$ is exact in R if and only if, $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$ in R. (CO4, K6)

Or

(b) State and prove the existence theorem for the convergence of the successive approximations. (CO4, K4)

20. (a) Let f be a real-value continuous function on the strip $S:|x-x_0|\leq a, |y|<\infty, (a>0)$ and suppose that f satisfies a Lipschitz condition on S with Lipschitz constant $K>0$. Then show that the successive approximations $\{\phi_k\}$ for the problem $y'=f(x, y)$, $y(x_0)=y_0$, exist on the entire interval $|x-x_0|\leq a$, and converges there to a solution ϕ. (CO5, K3)

Or

(b) Suppose f is a vector-valued function defined for (x, y) on a set S of the form $|x-x_0|\le a, |y-y_0|\le b, (a, b>0)$, or of the form $|x-x_0|\le a, |y|<\infty$, $(a>0)$. Prove that $\frac{\partial f}{\partial y_k}(k=1,...,n$ exists, is continuous on S, and there is a constant $K>0$ such that $\left|\frac{\partial f}{\partial y_k}(x, y)\right|\le K, (k=1,...n)$, for all (x, y) in S and f satisfies a Lipschitz condition on S with Lipschitz constant K. (CO5, K5)

R0064

Sub. Code
511104

M.Sc. DEGREE EXAMINATION, NOVEMBER – 2023

First Semester

Mathematics

ANALYTIC NUMBER THEORY

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 1 = 10)$

Answer **all** the following objective questions by choosing the correct option

1. 'If A is a non-empty set of positive integers, then A contains a smallest element' refers to (CO1, K2)

   (a) Induction principle

   (b) Division principle

   (c) Arithmetic principle

   (d) Well ordering principle

2. The value of $\phi(9)$ is (CO1, K2)

   (a) 4

   (b) 3

   (c) 6

   (d) 8

3. Which of the following is True? (CO2, K4)

(a) $\lim_{x\to\infty} \frac{1}{x} \Sigma_{n\le x}\mu(n) = 1$

(b) $\lim_{x\to\infty} \frac{1}{x} \Sigma_{n\le x}\mu(n) = 0$

(c) $\lim_{x\to\infty} \frac{1}{x} \Sigma_{n\ge x}\mu(n) = 1$

(d) $\lim_{x\to\infty} \frac{1}{x} \Sigma_{n\ge x}\mu(n) = 0$

4. Two lattice points (a,b) and (m,n) are mutually visible iff (CO2, K4)

(a) $a-m$ and $b-n$ are relatively prime

(b) $a+m$ and $b-n$ are relatively prime ψ

(c) $a-m$ and $b+n$ are relatively prime

(d) $a+m$ and $b+n$ are relatively prime

5. Chebychev's $\psi-$function is (CO3, K2)

(a) $\psi(x) = \Sigma_{n\ge x} \wedge (n)$ (b) $\psi(n) = \Sigma_{n\le x} \wedge (x)$

(c) $\psi(x) = \Sigma_{n\le x} \wedge (n)$ (d) $\psi(n) = \Sigma_{n\ge x} \wedge (x)$

6. Which of the following is correct? (CO3, K2)

(a) $\lim_{x\to\infty} \frac{\pi(x)\log x}{x} = 1$

(b) $\lim_{x\to\infty} \frac{\pi(x)\log x}{x} = 0$

(c) $\lim_{x\to\infty} \frac{x}{\pi(x)\log x} = 1$

(d) $\lim_{x\to\infty} \frac{x}{\pi(x)\log x} = 0$

7. The remainder when 41 divides $2^{20}-1$ is (CO4, K4)

(a) 1 (b) –1

(c) 2 (d) 0

8. The number of mutually incongruent solutions modulo 42 of the linear congruence $18x \equiv 0 \pmod{42}$ is (CO4, K4)

(a) 2 (b) 6

(c) 4 (d) 5

9. Which of the following is NOT True? (CO5, K5)

(a) 3 is a quadratic residue of 13

(b) 2 is a quadratic residue of 13

(c) 4 is a quadratic residue of 13

(d) 10 is a quadratic residue of 13

10. The Legendre's symbol is (CO5, K5)

(a) $(a/p) \equiv 1$ if a is a quadratic non residue of p

(b) $(a/p) \equiv -1$ if a is a quadratic residue of p

(c) $(a/p) \equiv 1$ if a is a quadratic residue of p

(d) $(a/p) \equiv -1$ if a is a quadratic non residue of p

Part B $(5 \times 5 = 25)$

Answer **all** the questions
not more than 500 words each.

11. (a) State and prove division algorithm. (CO1, K2)

Or

(b) If f and g are multiplicative then show that $f * g$ is multiplicative. (CO1, K2)

12. (a) State and prove Legendre's identity. (CO2, K4)

Or

(b) If $x \geq 1$, prove that $\Sigma_{n \leq x} \frac{1}{n} = \log x + C + O\left(\frac{1}{x}\right)$, where C is Euler's constant. (CO2, K4)

13. (a) State and Prove Abel's identity. (CO3, K2)

Or

(b) Show that $\lim_{x \to \infty}\left(\frac{M(x)}{x} - \frac{H(x)}{x \log x}\right) = 0$ (CO3, K2)

14. (a) Find the remainder when the sum. (CO4, K4)

1!+2!+3!+....+99!+100! is divided by 12.

Or

(b) Solve the linear congruence $17x \equiv 9 (\text{mod } 276)$. (CO4, K4)

15. (a) State and prove Euler's criterion. (CO5, K5)

Or

(b) Determine the solution of a quadratic congruence $x^2 \equiv 196 \pmod{1357}$. (CO5, K5)

Part C $(5 \times 8 = 40)$

Answer **all** the questions
not more than 1000 words each.

16. (a) Show that every positive integer $n > 1$ can be expressed uniquely as a product of primes apart from the order in which the factors occur. (CO1, K2)

Or

(b) State Generalized inversion formula and show its proof. (CO1, K2)

17. (a) State and Prove Euler's summation formula. (CO2, K4)

Or

(b) Show that the set of lattice points visible from the origin has density $\frac{6}{\pi^2}$. (CO2, K4)

18. (a) For every integer $n \geq 2$, show that $\frac{1}{6}\frac{n}{\log n} < \pi(n) 6. \frac{n}{\log n}$. (CO3, K2)

Or

(b) State Selberg's asymptotic formula and shor its proof. (CO3, K2)

19. (a) State and prove Chinese Remainder theorem. (CO4, K4)

Or

(b) Prove that the linear congruence $ax \equiv b \pmod{n}$ has a solution iff d / b, where $d = \gcd(a, n)$. (CO4, K4)

20. (a) State and prove Gauss lemma. (CO5, K5)

Or

(b) State quadratic reciprocity law and show its proof. (CO5, K5)

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R0065

Sub. Code
511505

M.Sc. DEGREE EXAMINATION, NOVEMBER – 2023

First Semester

Mathematics

Elective – OBJECT ORIENTED PROGRAMMING C++

(CBCS – 2022 onwards)

Time : 3 Hours   Maximum : 75 Marks

Part A $(10 \times 1 = 10)$

Answer **all** the following objective questions by choosing the correct option.

1. Stream is a ______________. (CO1, K1)

   (a) Group of non-printable character

   (b) Sequence of bytes

   (c) Set of errors

   (d) The flow of invalid characters

2. Which of the following is not a valid predefined object in C++? (CO1, K1)

   (a) cin   (b) cout

   (c) cput   (d) cerr

3. In a class, data members are also called as (CO2, K1)

   (a) Abstracts   (b) Attributes

   (c) Properties   (d) Dimensions

4. The data members and functions of a class in C++ are by default ______________. (CO2, K2)

(a) Protected (b) Private

(c) Public (d) Public and protected

5. Choose the right option. (CO3, K2)

string**x*, *y*:

(a) *x* is a pointer to a string, *y* is a string

(b) *y* is a pointer to a string, *x* is a string

(c) both *x* and *y* are pointers to string types

(d) *y* is a pointer to a string

6. What does the following statement mean? (CO3, K2)

int (*fp) (char*)

(a) Pointer to a pointer

(b) Pointer to an array of chars

(c) Pointer to function taking a char * argument and returns an int

(d) Function taking a char * argument and returning a pointer to int

7. Which type of function among the following shows polymorphism? (CO4, K4)

(a) Inline function

(b) Virtual function

(c) Undefined function

(d) Class member functions

8. Which one of the following can show polymorphism? (CO4, K5)

(a) Overloading || (b) Overloading &&

(c) Overloading << (d) Overloading + =

9. When a base class is privately inherited by a derived class public members of the base class become ______________ of the derived class. (CO5, K5)

(a) Private members (b) Protected members

(c) Public members (d) Not inherited

10. When a child class inherits traits from more than one parent class, this type of inheritance is called ______________ inheritance. (CO5, K6)

(a) Hierarchical (b) Hybrid

(c) Multilevel (d) Multiple

Part B $(5 \times 5 = 25)$

Answer **all** the questions not more than 500 words each.

11. (a) Explain the Basic concepts in oops. (CO1, K3)

Or

(b) Discuss the Manipulators in C++. (CO1, K3)

12. (a) Differentiate between the Class objects and class members. (CO2, K4)

Or

(b) How to Define and access member functions within a class. (CO2, K4)

13. (a) Explain the pointers and references. (CO3, K5)

Or

(b) Define *this* pointer, how to declare the *New* and *delete* operators? (CO3, K5)

14. (a) Discuss the Compile time polymorphism. (CO4, K6)

Or

(b) Appraise the Function overloading. (CO4, K6)

15. (a) How to declare Derived class in C++? (CO5, K5)

Or

(b) How does the Inheritance access specifier work in C++? (CO5, K6)

Part C (5 × 8 = 40)

Answer **all** the questions not more than 1000 words each.

16. (a) Brief about the data types and operators in c++. (CO1, K3)

Or

(b) State the Control loop statements and functions. (CO1, K3)

17. (a) Explain the Constructor and destructor. (CO2, K3)

Or

(b) How do the Friend, static and member functions work? (CO2, K3)

18. (a) Define Strings and explain the Dynamic constructors. (CO3, K4)

Or

(b) How to solve the Problems with pointer reference and copy constructor? Explain with example. (CO3, K4)

19. (a) Discuss the Operator overloading. (CO4, K5)

Or

(b) Explain the Overloading in unary and binary operators. (CO4, K5)

20. (a) Discuss the types of inheritance with example program. (CO5, K6)

Or

(b) Describe the Virtual and pure virtual functions. (CO5, K6)

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R0066

Sub. Code
511301

M.Sc. DEGREE EXAMINATION, NOVEMBER – 2023

Third Semester

Mathematics

CLASSICAL DYNAMICS

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 1 = 10)$

Answer **all** the following objective questions by choosing the correct option.

1. Total virtual work done on N-particle system is ———— (CO1, K1)

   (a) Zero

   (b) Maximum

   (c) Minimum

   (d) Neither maximum nor minimum

2. Kinetic energy of a particle of mass m is a ———— of the velocities. (CO1, K2)

   (a) Quadratic function

   (b) Homogeneous quadratic function

   (c) Linear function

   (d) Non-linear function

3. In Lagrange's equation if there are N number of particles and so the generalized coordinates are (CO2, K2)

(a) $n = N - k$ (b) $n = 3N - k$

(c) $n = 3N$ (d) $n = 3n - k$

4. In case of simple pendulum, the normal restoring force is ―――――― (CO2, K2)

(a) $-mgl$ (b) $-mgl\cos\theta$

(c) $-mgl\sin\theta$ (d) $mgl\sin\theta$

5. The action integral must be a ―――――― value for actual path. (CO3, K3)

(a) Real (b) Stationary

(c) Maximum (d) Minimum

6. All the virtual displacements δq_j are ―――――― (CO3, K3)

(a) Zero (b) Dependent

(c) Independent (d) None of these

7. $\int_{t_1}^{t_2} \delta L\, dt = 0$ is called ―――――― (CO4, K4)

(a) Hamilton's Principle

(b) Lagrangian Principle

(c) Minimal Integral

(d) Liouville's Principle

8. The conserved quantity in the system that has transactional symmetry is ———— (CO5, K5)

(a) Momentum (b) Area

(c) Velocity (d) Displacement

9. The point transformation is also known as ———— (CO5, K6)

(a) Contact transformation

(b) Functional transformation

(c) Non-functional transformation

(d) None of these

10. The bridge between Classical Mechanics is provided by ———— (CO5, K6)

(a) Lagrange brackets

(b) Poisson brackets

(c) Jacobi's identity

(d) Hamilton's identity

Part B (5 × 5 = 25)

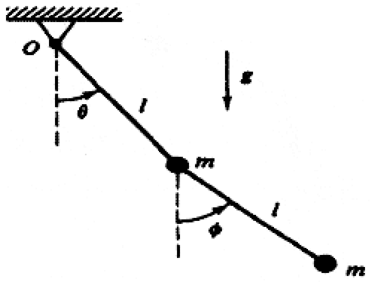
Answer **all** the questions not more than 500 words each.

11. (a) A particle of mass m is suspended by a massless wire of length $r = a + b\cos\omega t$, $(a < b > 0)$ to form a spherical pendulum. Find the equations of motion. (CO1, K3)

Or

(b) State and prove Konig's theorem. (CO2, K3)

12. (a) A double pendulum consists of two particles suspended by massless rods, as shown below. Assuming that all motion takes place in a vertical plane. Find the differential equations of motion. (CO2, K3)



Or

(b) Write a note on Routhian function. (CO3, K4)

13. (a) Describe and solve the geodesic problem. (CO4, K5)

Or

(b) State and solve the brachistochrone problem. (CO4, K5)

14. (a) State and prove Jacobi's theorem. (CO5, K6)

Or

(b) Analyze the Kepler problem by using the Hamilton-Jacobi method. (CO4, K6)

15. (a) Find K-H and the generating functions of the transformation $Q = q - tp + \frac{1}{2}gt^2$, $p = p - gt$. (CO5, K6)

Or

(b) State and prove Poisson's theorem. (CO5, K6)

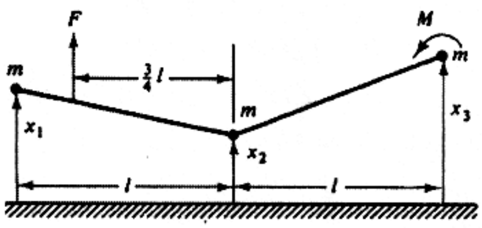
Part C (5 × 8 = 40)

Answer **all** the questions not more than 1,000 words each.

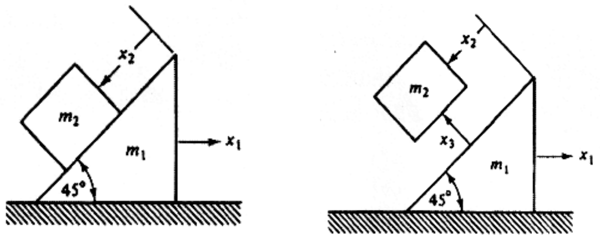
16. (a) Explain about principle of virtual work with examples. (CO2, K3)

Or

(b) Three particles are connected by two rigid rods having a joint between them to form the system shown below. A vertical force F and a moment M are applied as shown. The configuration of the system is given by the ordinary coordinates (x_1, x_2, x_3) or by the generalized coordinates (q_1, q_2, q_3), where $x_1 = q_1 + q_2 + \frac{1}{2}q_3$, $x_2 = q_1 - q_3$, $x_3 = q_1 - q_2 + \frac{1}{2}q_3$. Find the generalized forces Q_1, Q_2 and Q_3. (CO3, K4)



17. (a) A block of mass m_2 can slide on another block of mass m_1 which, in turn, slides on a horizontal surface, as shown below. Using x_1 and x_2 as coordinates, obtain the differential equation of motion. Solve for the accelerations of the two blocks as they move under the influence of gravity, assuming that all surfaces are frictionless. Find the force of interaction between the blocks. (CO4, K4)



Or

(b) (i) Describe Liouville's system. (CO4, K5)

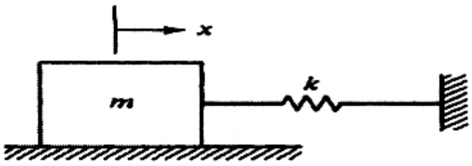
(ii) Obtain the energy integral for a conservative system.

18. (a) Describe and derive the principle of least action. (CO4, K6)

Or

(b) Derive Hamilton's equation for a holonomic system. (CO5, K5)

19. (a) Use Hamilton-Jacobi method to solve for a simple mass spring system figure given below. (CO5, K6)



Or

(b) (i) Write a short note on Pfaffian differential equation. (CO5, K4)

(ii) State and prove Stackel's theorem.

20. (a) Obtain the Homogeneous canonical transformation and point transformations. (CO5, K6)

Or

(b) Explain about Lagrange and Poisson brackets. (CO5, K6)

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R0067

Sub. Code
511302

M.Sc. DEGREE EXAMINATION, NOVEMBER – 2023

Third Semester

Mathematics

TOPOLOGY

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 1 = 10)$

Answer **all** the following objective questions by choosing the correct option.

1. If X is any set and the collection of all subsets of X is a topology on X then the topology is called ———— (CO1, K1)
   (a) Indiscrete topology
   (b) Trivial topology
   (c) Discrete topology
   (d) Complement topology

2. Choose subspace topology ———— (CO1, K2)
   (a) $\Im_y = \{Y \cap U \mid U \in \Im\}$
   (b) $\Im_y = \{Y \cup U \mid U \in \Im\}$
   (c) $\Im_U = \{Y \cap U \mid U \in \Im\}$
   (d) $\Im_U = \{Y \cup U \mid U \in \Im\}$

3. Choose the diameter of A is defined to be the number, if A is bounded and nonempty ————. (CO2, K2)
   (a) $\overline{d}(x, y) = \min\{d(x, y), 1\}$
   (b) $\overline{d}(x, y) = \max\{d(x, y), 1\}$
   (c) $\overline{d}(x, y) = \max\{d(x, y), 0\}$
   (d) $\overline{d}(x, y) = \min\{d(x, y), 0\}$

4. Identity, the image of a connected space under a continuous map is ————————. (CO2, K1)

(a) Continuous (b) Compact

(c) Connected (d) Disconnected

5. Choose, every closed interval in R is ———————— (CO3, K1)

(a) Countable (b) Uncountable

(c) Empty (d) Compact

6. The space X is said to be ———————— if every sequence of points of X has a convergent subsequence. (CO3, K1)

(a) Locally compact (b) Separation

(c) Limit point (d) Sequentially compact

7. Name the subset A of a space X which is ———————— in X if $\overline{A} = X$. (CO4, K1)

(a) Dense (b) Separable

(c) Countable (d) Basis

8. Every regular space with a countable basis is ————————. (CO4, K1)

(a) Normal (b) Regular

(c) Metrizable (d) Countable

9. Name the 1-manifold is ————————. (CO5, K1)

(a) Surface (b) Curve

(c) Support (d) Unity

10. Every manifold is ————————. (CO5, K1)

(a) Normal (b) Regular

(c) Separable (d) Compact

Part B (5 × 5 = 25)

Answer **all** the questions not more than 500 words each.

11. (a) Let X be an ordered set in the order topology. Let Y be a subset of X that is convex in X. Then prove that the order topology on Y is the same as the topology Y inherits as a subspace of X. (CO1, K5)

Or

(b) Let A be a subset of the topological space X; let A' be the set of all limit points of A. Then prove that $\overline{A} = A \cup A'$. (CO1, K5)

12. (a) State and Prove the sequence lemma. (CO2, K5)

Or

(b) Prove that the union of a collection of connected subspaces of X that have a point in common is connected. (CO2, K5)

13. (a) Show that every compact subspace of a Hausdorff space is closed. (CO3, K2)

Or

(b) Show that Compactness implies limit point compactness, but not conversely. (CO3, K2)

14. (a) Prove that every metrizable space is normal. (CO4, K5)

Or

(b) Prove that every compact Hausdorff space is normal. (CO4, K5)

15. (a) Let $A \subset X$; let $f : A \to Z$ be continuous map of A into the Hausdorff space Z. Then prove that there is at most one extension of f to a continuous function $g : \overline{A} \to Z$. (CO5, K5)

Or

(b) Let X be a set; let $\mathcal{D}$ be a collection of subsets of X that is maximal with respect to the finite intersection property. Then prove that any finite intersection of elements of $\mathcal{D}$ is an element of $\mathcal{D}$. (CO5, K5)

Part C (5 × 8 = 40)

Answer **all** the questions not more than 1000 words each.

16. (a) Let X be a topological space. Suppose that C is a collection of open sets of X such that for each open set U of X and each x in U, there is an element C of C such that $x \in C \subset U$. Then prove that C is a basis for the topology of X. (CO1, K5)

Or

(b) Show that let Y be a subspace of X. Then a set A is closed in Y if and only if it equals the intersection of a closed set of X with Y. (CO1, K2)

17. (a) Show that a finite Cartesian product of connected space is connected. (CO2, K2)

Or

(b) Prove that if L is a linear continuum in the order topology, then L is connected, and so are intervals and rays in L. (CO2, K5)

18. (a) Prove that the product of finitely many compact spaces is compact. (CO3, K5)

Or

(b) State and Prove the Lebesgue number lemma. (CO3, K5)

19. (a) Show that every well-ordered set X is normal in the order topology. (CO4, K2)

Or

(b) State and Prove Urysohn metrization theorem. (CO4, K5)

20. (a) State and Prove Tychonoff theorem. (CO5, K5)

Or

(b) Prove that if X is a compact m-manifold, then X can be imbedded in R^N for some positive integer N. (CO5, K5)

———————

R0068

Sub. Code
511303

M.Sc. DEGREE EXAMINATION, NOVEMBER – 2023

Third Semester

Mathematics

CALCULUS OF VARIATIONS AND INTEGRAL EQUATIONS

(CBCS – 2022 onwards)

Time : 3 Hours   Maximum : 75 Marks

Part A   (10 × 1 = 10)

Answer **all** the following objective questions by choosing the correct option.

1. The extremal of the functional $J[y] = \int_a^b (x-y)^2 \, dx$ is (CO1, K1)

   (a) $y = x^2$ (b) $x = y^2$

   (c) $y = x$ (d) $y = 0$

2. The operators δy and $\frac{d}{dx}$ are commutative if (CO1, K2)

   (a) $\frac{d}{dx} \cdot \delta y = \delta \frac{dy}{dx}$ (b) $\frac{d}{dx} \delta x = \delta \frac{dx}{dy}$

   (c) $\frac{d}{dx} \cdot \delta y = \delta y \frac{d}{dx}$ (d) $\frac{d}{dx} \delta x = \delta x \frac{dx}{dy}$

3. Identify the Beltrami identity (CO2, K2)

(a) $F - y'' \dfrac{\partial F}{\partial y'} = 0$

(b) $F - y' \dfrac{\partial F}{\partial y'} = 0$

(c) $F - y'' \dfrac{\partial F}{\partial y'} =$ constant

(d) $F - y' \dfrac{\partial F}{\partial y'} =$ constant

4. The functional $I(y(x)) = \int_a^b (y^2 + y'^2 - 2y\sin x)\,dx$ has the following extremal with c_1 and c_2 are arbitrary constants. (CO2, K2)

(a) $y = c_1 e^{2x} + c_2 e^{-2x} + \frac{1}{2}\sin x$

(b) $y = c_1 e^{2x} + c_2 e^{-2x} + \frac{1}{2}\cos x$

(c) $y = c_1 e^{x} + c_2 e^{-x} + \frac{1}{2}\sin x$

(d) $y = c_1 e^{x} + c_2 e^{-x} - \frac{1}{2}\sin x$

5. Which of the following is the Hankel transform of e^{-x}? (CO1, K2)

(a) $(1 - p^2)^{\frac{3}{2}}$ (b) $(1 + p^2)^{\frac{3}{2}}$

(c) $(1 - p^2)^{\frac{2}{3}}$ (d) $(1 + p^2)^{\frac{2}{3}}$

6. If $J_n(x)$ denotes the Bessel function of the first kind, then $\frac{d}{dx}[x^{-n} J_n(x)] =$ (CO3, K3)

(a) $x^{-n} J_{n+1}(x)$ (b) $-nx^{n-1} J_n(x)$

(c) $-x^{-n} J_{n+1}(x)$ (d) $-nx^{-n} J_{n+1}(x)$

7. Which one of the following is true? (CO2, K2)

(a) Two functions Φ and ψ is orthogonal if $\langle \psi, \Phi \rangle = \| \psi + \Phi \|$

(b) A function ψ is normalized if $\| \psi \| = 0$

(c) Two functions Φ and ψ is orthogonal if $\langle \psi, \Phi \rangle = 1$

(d) A function ψ is normalized if $\| \psi \| = 1$

8. Which one of the following is true? (CO2, K3)

(a) with usual notation, if $D(\lambda) = 0$, then the inhomogeneous has unique solution

(b) If $D(\lambda) = 0$, then the inhomogeneous has infinitely many solution

(c) If $D(\lambda) = 0$, then the inhomogeneous has no solution

(d) none of these

9. Which one of the following is false? (CO3, K2)

(a) with usual notation, every zero of $D(\lambda)$ is the pole of the resolvent kernel

(b) The resolvent kernel is an analytic function of λ, regular at least inside the circle $\lambda < 1/B$

(c) The resolvent kernel is a quotient of two polynomials of n^{th} degree in λ

(d) None of these

10. Which one of the following is inhomogeneous Fredholm equation? (CO3, K3)

(a) $g(s) = f(s) + \lambda \int \Gamma(s,t;\lambda) f(t)\, dt$

(b) $g(s) = f(s) + \lambda \int K(s,t)\, g(t)\, dt$

(c) $g(s) = \lambda \int K(s,t)\, g(t)\, dt$

(d) None of these

Part B (5 × 5 = 25)

Answer **all** the questions not more than 500 words each.

11. (a) Find the extremal of the functional $J[y] = \int_1^2 \frac{\sqrt{1+(y^1)^2}}{x} dx,\ y(1) = 0\ ,\ y(2) = 1\ .$ (CO2, K3)

Or

(b) Derive the Euler's equation for the functional $J[z] = \int\int_R \left[\frac{\partial z^2}{\partial x} - \frac{\partial z}{\partial y}\right]^2 dx\, dy\ .$ (CO3, K4)

12. (a) Find the shortest smooth plane curve joining two distinct points (x_1, y_1) and (x_2, y_2). (CO3, K3)

.Or

(b) Find the extremals of the functional $\int_{x_0}^{x_1} \frac{y'^2}{x^3} dx\ .$

(CO4, K3)

13. (a) If $\widetilde{f}(p)$ and $\widetilde{g}(p)$ are the Hankel transform of the functions $f(x)$ and $g(x)$ respectively. Prove that $\int_0^\infty x f(x) g(x) dx = \int_0^\infty p\widetilde{f}(p)\widetilde{g}(p) dp$. (CO4, K5)

Or

(b) Find the Hankel transform of $f(x) = \begin{cases} x^n & 0 < x < a \\ 0 & x > a \end{cases}$, taking $xJ_n(px)$ as the Kernel. (CO3, K4)

14. (a) Write a short note on different types of Kernels. (CO4, K4)

Or

(b) Find the eigenvalues and eigenfuntions of the homogeneous integral equation $g(s) = \lambda \int_1^2 [st + (1/st)] g(t) dt$. (CO4, K5)

15. (a) Solve the integral equation $g(s) = 1 + \lambda \int_0^\pi [\sin(s+t)] g(t) dt$. (CO5, K6)

Or

(b) Evaluate the resolvent for the integral equation $g(s) = f(s) + \lambda \int_0^1 (s+t) g(t) dt$. (CO5, K6)

Part C $(5 \times 8 = 40)$

Answer **all** the questions not more than 1,000 words each.

16. (a) Show that a necessary condition for the functional $J(z) = \iint_R F(x, y, z, z_x, z_y)\,dx\,dy$ to have an extremum for a given function $z(x,y)$ is that $z(x,y)$ satisfies the equation $F_z - \frac{\partial}{\partial x}(F_{z_x}) - \frac{\partial}{\partial y}(F_{z_y}) = 0$. (CO3, K5)

Or

(b) Prove that the derivation of the variation with respect to an independent variable is the same as the variation of the deviation. (CO4, K5)

17. (a) Explain Brachistochrone problem. (CO4, K5)

Or

(b) Explain Dido's problem. (CO5, K4)

18. (a) Prove that the Hankel transform of $\frac{d^2 f}{dx^2} + \frac{1}{x}\frac{df}{dx} - \frac{n^2}{x^2} f$ is $-p^2 \tilde{f}_n(p)$. (CO5, K6)

Or

(b) If $\tilde{f}(p)$ is the Hankel transform of the function $f(r)$, prove that $f(r) = \int_0^\infty \tilde{f}(p)\, p J_n(pr)\,dp$. (CO4, K6)

19. (a) Find the resolvent kernel for the integral equation $g(s) = f(s) + \lambda \int_{-1}^{1} (st + s^2t^2)\, g(t)\, dt$. (CO5, K6)

Or

(b) Solve the integral equation $g(s) = f(s) + \lambda \int_{0}^{1} (1 - 3st)\, g(t)\, dt$. (CO4, K5)

20. (a) State and prove Fredholm's second theorem. (CO5, K6)

Or

(b) Find the Neumann series for the solution of the integral equation $g(s) = (1 + s) + \lambda \int_{0}^{s} (s - t)\, g(t)\, dt$. (CO5, K6)

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R0069

Sub. Code
511518

M.Sc. DEGREE EXAMINATION, NOVEMBER – 2023

Third Semester

Mathematics

Elective — OPTIMIZATION TECHNIQUES

(CBCS – 2022 onwards)

Time : 3 Hours — Maximum : 75 Marks

Part A $(10 \times 1 = 10)$

Answer **all** the following objective questions by choosing the correct option.

1. Which of the following is Hessian matrix of $f(x, y) = x^2 - y^2$ at $(0, 0)$? (CO1, K1)

   (a) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

   (c) $\begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

2. The matrix $[A] = \begin{bmatrix} 4 & 2 & -4 \\ 2 & 4 & -2 \\ -4 & -2 & 4 \end{bmatrix}$ is ———— (CO1, K1)

   (a) Positive definite (b) Positive semidefinite

   (c) Negative definite (d) Indefinite

3. Which of the following is maximum value of the function $f(x, y) = x + 3y$? (CO1, K2)

Subject to

$-4x + 3y \leq 12$

$x + y \leq 7$

$x - 4y \leq 2$

$x \geq 0,\ y \geq 0.$

(a) $x = 0,\ y = 0$ (b) $x = \frac{1}{4},\ y = \frac{40}{7}$

(c) $x = \frac{9}{7},\ y = \frac{40}{7}$ (d) $x = 1,\ y = 1$

4. Let S be a closed convex polyhedron. Then the minimum of a linear function over S is attained at which point of S? (CO2, K2)

(a) Interior point of S

(b) Exterior point of S

(c) Extreme point of S

(d) Set of points of S in the convex set that lie on a line segment joining two other points of the set

5. Which of the following is Direct search method? (CO2, K3)

(a) Steepest method

(b) Newton's method

(c) Marquardt method

(d) Powell's method

6. Which of the following method is based on generating a sequence of improved approximations to the minimum, each derived from the preceding approximation? (CO3, K3)

(a) Random Jumping Method

(b) Random Walk Method

(c) Powell's Method

(d) Unvariate Method

7. Which of the following method is called sequential linear programming method? (CO3, K4)

(a) Complex method (b) Zoutendijk method

(c) Rosen's method (d) Cutting plane method

8. Find a method which transform the basic optimization problem into alternative formulations such that numerical solutions are sought by solving a sequence of unconstrained minimization problems. (CO4, K5)

(a) Penalty function method

(b) Cutting plane method

(c) Projected Lagrangian method

(d) Rosen's method

9. Which of the following is a non-linear programming problem? (CO5, K5)

(a) All-integer problem

(b) Mixed-integer problem

(c) Zero-one problem

(d) Polynomial programming problem

10. In Gomary's cutting plane method, the feasible integer solution of the problem are denoted by dots. These points are called the (CO5, K6)

(a) integer optimal points

(b) integer comer points

(c) integer lattice points

(d) integer initial points

Part B (5 × 5 = 25)

Answer **all** the questions not more than 500 words each.

11. (a) State and prove the necessary condition for the relative minimum of a function of a single variable. (CO1, K3)

Or

(b) Find the values of x, y and z that maximize the function $f(x, y, z) = \frac{6xyz}{x + 2y + 2z}$, when x, y and z are restricted by the relation $xyz = 16$. (CO2, K4)

12. (a) Prove that the feasible region of a linear programming problem is convex. (CO3, K5)

Or

(b) Solve Maximize $f = x - 4y$ (CO4, K5)

Subject to

$x - y \geq -4$

$4x + 5y \leq 45$

$5x - 2y \leq 20$

$5x + 2y \geq 10$

$x \geq 0$, is unrestricted to sign.

13. (a) Prove that the gradient vector represents the direction of steepest ascent. (CO4, K4)

Or

(b) Solve the following equations using the steepest descent method (two iterations only) with the starting point. $X_1 = \{0, 0, 0\}$.

$2x_1 + x_2 = 4$, $x_1 + 2x_2 + x_3 = 8$, $x_2 + 3x_3 = 11$ (CO5, K5)

14. (a) Write the characteristics of a constrained problem. (CO4, K4)

Or

(b) Write the algorithm of sequential linear programming method. (CO4, K5)

15. (a) Write the solution procedure of Gomory's method for mixed-integer programming problems. (CO5, K5)

Or

(b) Solve the following problem using Gomory's cutting plane method : (CO5, K5)

Maximize $f = x_1 + 2x_2$

Subject to

$x_1 + x_2 \le 7$

$2x_1 \le 11,\ 2x_2 \le 7$

$x_i \ge 0$ and integer, $i = 1, 2$.

Part C (5 × 8 = 40)

Answer **all** the questions not more than 1000 words each.

16. (a) Find the dimensions of a cylindrical tin (with top and bottom) made up of sheet metal to maximize its volume such that the total surface area is equal to $A_0 = 24\ \pi$. (CO3, K6)

Or

(b) Consider the following problem : (CO3, K5)

Minimize $f = (x_1 - 2)^2 + (x_2 - 1)^2$

Subject to

$2 \ge x_1 + x_2$

$x_2 \ge x_1^2$ using

Kuhn-Tucker condition, find which of the following vectors are local minima :

$X_1 = \begin{Bmatrix} 1.5 \\ 0.5 \end{Bmatrix}\ X_2 = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix},\ X_3 = \begin{Bmatrix} 2 \\ 0 \end{Bmatrix}.$

17. (a) By using two-phase simplex method solve minimize $f = 2x_1 + 3x_2 + 2x_3 - x_4 + x_5$ (CO5, K6)

subject to the constraints

$3x_1 - 3x_2 + 4x_3 + 2x_4 - x_5 = 0$

$x_1 + x_2 + x_3 + 3x_4 + x_5 = 2$

$x_i \geq 0, i =$ 1 to 5.

Or

(b) By using quadratic programming method, solve minimize $f = -4x_1 + x_1^2 - 2x_1x_2 + 2x_2^2$ (CO5, K5)

subject to

$2x_1 + x_2 \leq 6$

$x_1 - 4x_2 \leq 0$

$x_1 \geq 0, x_2 \geq 0$

18. (a) By using steepest descent method solve Minimize $f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$ starting from the point $X_1 = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$. (CO5, K5)

Or

(b) Write detailed procedure of the Random walk method. (CO5, K6)

19. (a) Solve minimize $f(x_1, x_2) = x_1 - x_2$ subject to $g_1(x_1, x_2) = 3x_1^2 - 2x_1x_2 + x_2^2 - 1 \leq 0$ using the cutting plane method. Take the convergence limit in step 5 as $\varepsilon = 0.02$. (CO5, K6)

Or

(b) State and prove the convergence of the interior penalty function method. (CO5, K6)

20. (a) By using Gomory's cutting plane method, solve Minimize $f = -3x_1 - 4x_2$ (CO4, K6)

subject to

$3x_1 - x_2 + x_3 = 12$

$3x_1 + 11x_2 + x_4 = 66$

$x_i \geq 0$ i = 1, 2, 3, 4 and a_{11} x_i are integers.

Or

(b) Solve the following LP problem using the branch and bound method : (CO5, K6)

Maximize $f = 3x_1 + 4x_2$

Subject to

$7x_1 + 11x_2 \leq 88$

$3x_1 - x_2 \leq 12$

$x_1 \geq 0$, $x_2 \geq 0$ and x_1 and x_2 are integers.